Αλγόριθμοι Δικτύων και Πολυπλοκότητα
Το πρόβλημα Routing and Path Coloring και οι εφαρμογές του σε πλήρως οπτικά δίκτυα

Άρης Παγουρτζής

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Optical Fibers

• High transmission rate
• Low bit error rate
• The bottleneck lies in converting an electronic signal to optical and vice versa

All-Optical Networks

• All physical connections are optical
• Multiplexing is achieved through wavelength division multiplexing (WDM): in each fiber multiple colors are used
• Switching on routers is done passively and thus more effectively (no conversion from electrical to optical)
• Two network nodes communicate using one light beam: a single wavelength is used for each connection
Graph Representation

• All physical links are represented as graph edges
• Communication among nodes is indicated by paths
• Paths are assigned colors (wavelengths)
• Overlapping paths (i.e. sharing at least one edge) are assigned different colors
Graph Topologies
Graph Coloring (GC)

- Input: Graph $G$
- Feasible solution: Coloring of $V$ using different colors for adjacent vertices
- Goal: Minimize the number of colors used, i.e. find chromatic number $\chi(G)$

- NP-hard
- There is no approximation algorithm of ratio $n^{\varepsilon}$ for some $\varepsilon > 0$ (polyAPX-hard)
- Lower bound for $\chi(G)$: order (size) $\omega$ of maximum clique of $G$
Edge Coloring (EC)

• Input: Graph $G$

• Feasible solution: Coloring of $E$ using different colors for adjacent edges

• Goal: Minimize the number of colors used, i.e. find chromatic index $\chi'(G)$

• Lower bound for $\chi'(G)$: maximum degree $\Delta(G)$

• [Vizing’64]: between $\Delta(G)$ and $\Delta(G)+1$ (simple graphs)
  between $\Delta(G)$ and $3\Delta(G)/2$ (multigraphs)

• [Holyer’80]: NP-complete whether $\Delta(G)$ or $\Delta(G)+1$

• 4/3 -approximable in simple graphs and multigraphs

• Best possible approximation unless P=NP
Path Coloring (PC)

- Input: Graph $G$, set of paths $P$
- Feasible solution: Coloring of paths s.t. overlapping paths are not assigned the same color
- Goal: Minimize the number of colors used
- Lower bound: maximum load $L$
- We can reduce it to GC by representing paths as vertices and overlapping paths as edges (*conflict graph*)
- Improved lower bound: order $\omega$ of the maximum clique of the conflict graph
Path Coloring (PC)

- Corresponding decision problem is NP-complete
- In general topologies the problem is poly-APX-hard
- Proof: Reduction of GC to PC in meshes [Nomikos’96]
Chain PC

- Solved optimally in polynomial time with exactly $L$ colors

Ring PC

- Also known as Arc Coloring
- NP-complete [GJMP 80]
- Easily obtained appr. factor 2:
  Remove edge $e$ and color resulting chain. Color all remaining paths that pass through $e$ with new colors (one for each path)
  
  \[
  SOL_c \leq L
  \]
  
  \[
  SOL \leq SOL_c + L \leq 2 \cdot OPT
  \]

- W. K. Shih, W. L. Hsu: appr. factor 5/3
- I. Karapetian: appr. factor 3/2
- Idea: Use of maximum clique of conflict graph
Ring PC

- V. Kumar: With high probability appr. factor 1.36
- Idea: Use of *multicommodity flow problem*
Star PC

NP-completeness: Reduction of EC to Star PC

Approximation ratio: at least 4/3
Star PC: Approximability

Reduction of Star PC to EC in multigraphs

Approximation ratio: 4/3
Tree PC

Recursive Algorithm

if tree is a star then color it approximately
else
  – Subdivide the tree by “breaking” one of its internal edges
  – Color the resulting subtrees
  – Join sub-instances by rearranging colors
Tree PC (ii)

\[ P_1 = \{ q = p \cap T_1 \mid p \in P \} \]
\[ P_2 = \{ q = p \cap T_2 \mid p \in P \} \]

Approximation ratio equal to the one achieved by the approximate Star PC algorithm, thus 4/3
Bounded Degree Tree PC

- Trees of bounded degree are reduced by the above reduction to multigraphs of bounded size

- EC in bounded size multigraphs can be solved optimally in polynomial time
Generalized Tree \((S,d)\) PC

- Finite set of graphs \(S\)
- Tree of degree at most \(d\)
- Optimally (exactly) solvable in polynomial time

Idea:
- Since graphs are finite, coloring can be done in \(|P| f(S,d)\)
- Recursive algorithm, color rearrangement
- Application: Backbone Networks of customized LANs
Directed Graphs
PC in directed graphs

- **D-Chain PC**: Reduced to two undirected instances
- **D-Ring PC**: As above
- **D-TreePC**: Approximated within a $5/3$ factor. Least possible factor is $4/3$, though the algorithm known is the best possible among all greedy algorithms [Erlebach, Jansen, Kaklamanis, Persiano’97]
- **D-TreePC**: Not solved optimally in bounded degree trees
Routing and Path Coloring (RPC)

- Input: Graph $G$, set of requests $R \subseteq V^2$
- Feasible solution: Routing of requests in $R$ via a set of paths $P$ and color assignment to $P$ in such a way that overlapping paths are not assigned the same color
- Goal: Minimize the number of colors used

In acyclic graphs (trees, chains) RPC and PC coincide
Ring RPC

“Cut-a-link” technique  [Raghavan-Upfal’94]
• Pick an edge e
• Route all requests avoiding edge e
• Solve chain instance with $L$ colors

Thm: The above is a 2-approximation algorithm
Proof: $L \leq 2 L_{opt} \leq 2 OPT$

V. Kumar: 1.68-approximation with high probability
Tree of Rings RPC

Approximation ratio 3
RPC in (bi)directed topologies

• In acyclic topologies PC and RPC coincide.

• In rings there is a simple 2-approximation algorithm.

• In trees of rings the same as before technique gives approximation ratio $10/3 (=2 \times 5/3)$.
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